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DISCUSSION

Comments on: Latent Markov models: a review of the general framework for the analysis of longitudinal data with covariates

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1 Introduction

We heartily congratulate Bartolucci, Farcomeni, and Pennoni for their review of latent Markov (LM) models (Bartolucci et al. 2014). Not only have they provided a succinct and thorough guide that will benefit researchers seeking to employ LM models for many years to come, but they also have offered their suggestions on a number of further developments to extend the basic LM framework that provide direction for future research. In this comment, we would like to pick up on the suggested developments concerning more flexible temporal structures by highlighting two approaches that have proved useful in related domains; we do not view this as criticism of the proposed LM framework but rather as extensions of the review that are advantageous in terms of parsimony and scalability.

2 Parsimonious extensions of the first-order model

Bartolucci et al. (2014) note that the assumption that the LM model is first-order can sometimes be too restrictive, citing the case where the holding time in one or more states is not memoryless (i.e., geometric) as required by the first-order Markov model. They suggest generalizing the basic first-order Markov model to higher orders to allow for memory. A challenge with this approach, however, is that the number of parameters explodes as one moves from first-order to higher-order Markov models.

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An approach that has been developed to deal with this challenge is the generalized Markov (GM) model of McShane et al. (2013). The GM model accommodates non-memoryless (i.e., non-geometric) holding times whilst remaining computationally feasible, parsimonious, and easily estimable by assuming parametric holding time distributions for each state. The key to this approach involves embedding the GM chain on the original state space into a first-order Markov chain on an augmented state space. While the augmented state space is much larger than the original state space, the augmented parameter space associated with the augmented state space is quite parsimonious (e.g., the entries in the transition probability matrix of the augmented state space are mostly zeroes).

The setting of McShane et al. (2013) was similar to the LM setting in that the goal was to model a response $\tilde{\boldsymbol{Y}}$ given covariates $\tilde{\boldsymbol{X}}$. However, in their case there were no latent (or hidden) states $\boldsymbol{U}$; instead, a GM model was assumed for $\tilde{\boldsymbol{Y}}$ and the goal was to predict unknown $\tilde{\boldsymbol{Y}}$ out-of-sample given covariates $\tilde{\boldsymbol{X}}$ [in their case $\boldsymbol{Y}^{(t)}$ was a scalar categorical response variable with three levels so that $r = 1$ in the notation of Bartolucci et al. (2014)].

To generalize the GM model to the LM setting, the formulation presented in Section 4.1.2 of Bartolucci et al. (2014) would require only a few minor adjustments. First, the model for the initial probabilities would remain the same. Second, the model for the transitions would remain the same except that there would be no self-transitions. Third, self-transitions would be governed instead by state-specific parametric holding time distributions; the parameters of these holding time distributions could, if desired, depend on the covariates. The transition matrix and holding time distributions would combine to form the augmented state space and augmented transition matrix as outlined in McShane et al. (2013). Inference would follow just as in Section 6 of Bartolucci et al. (2014) except now there would also be parameters for the holding time distributions.

The major difference between the setting of McShane et al. (2013) and the latent setting would be in how to choose parametric forms for the holding time distributions. While McShane et al. (2013) were able to observe holding times using their in-sample data and thus could model these holding times directly, this approach is clearly not possible in the latent setting. Since the default first-order Markov model assumes geometric holding times, perhaps a reasonable approach is to first consider relatively simple generalizations of the geometric distribution (e.g., the beta-geometric, the negative binomial, and the beta-negative binomial) as holding time distributions and test whether the estimated parameters imply a strong divergence from the basic geometric model. As these distributions are relatively flexible, they could likely accommodate a wide variety of shapes for the holding times in each latent state and would thus offer an advance over current first-order Markov models without the concomitant explosion in the number of parameters.

3 Continuous-time extensions for unequally spaced time intervals

Many social, psychological, and biological processes evolve continuously in time. Consequently, there may not always be a substantively compelling reason to prefer one time interval to another when using a discrete-time Markov model. However, the specification of the time interval is a non-trivial consideration. For example, the LM model fit to the criminal dataset in Section 3.3 of Bartolucci et al. (2014) appears to fit well when $T = 6$ five-year age bands covering a range of thirty years are adopted. However, two limitations with this approach should be noted. First, the observations were not collected at the same time for each panel member but were instead aggregated within multiple five-year periods; inaccuracies introduced by this high-level aggregation are potentially considerable but cannot be quantified on the basis of the fitted LM model. Second, little can be said about transition processes at a more fine-grained level of analysis; for example, while the first-order Markov model may be sufficient to model the holding times in each latent state with these five-year bands, it may not be adequate for a different time window. In sum, any results obtained may be highly dependent on potentially arbitrary choices in the selection of the time window.

Dependence on the choice of a time window can be avoided by treating latent processes in continuous time. As an alternative to following a fixed time schedule that may miss or mask important shifts in the latent change process of each time series, one can time data collection in such a way that latent switches have a high probability of occurring. For example, Böckenholt (2005) presented an analysis of an experience-sampling study where participants were asked to report their current affective state at five randomly selected time points during the course of a day over a period of two weeks. Instead of discretizing the data at, for example, the hourly level, he fit a continuous-time LM model that explicitly accounted for the random time intervals between the self-reports. By accommodating duration-dependent holding times, this approach provided useful insights about chronometric features of emotional states that did not depend on the selection of a particular time window.

4 Conclusion

A stronger emphasis on the time scale of a latent switching process raises important questions concerning how to improve the accuracy of time-structured inferences. For example, multiple observations within a day may be sufficient to accurately describe psychological processes on an hourly basis; on the other hand, observations taken over the course of different weeks, months, or years provide information at a cruder time scale and thereby potentially limit the types of change processes that can be studied. In view of the expanding availability of longitudinal data with data structures that may include multiple time units, modeling strategies for latent change processes should become correspondingly more informative about the underlying time dependencies. We highlighted approaches that go beyond the basic first-order Markov model specification and allow for continuous-time treatments, but much work remains to be done. Future developments would benefit from considering the integration of momentary, short-term, and long-term change processes and their facilitating or inhibiting effects at different time scales. The general LM model presented by Bartolucci et al. (2014) is a major step in this direction.

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